Math 270: Differential Equations Day 11 Part 2

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 1)

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 1)

- In the last section, we were trying to guess a particular solution y_p to the **non-homogeneous** 2nd-order linear DEs w/constant coefficients $ay'' + by' + cy = f(t)$ but the right side $f(t)$ consisted of only one term?

Like: $y'' + 2y' + 5y = t^2 e^{3t} \sin t$

- What if $f(t)$ has more than one term being added or subtracted together?

Like: $y'' + 2y' + 5y = t^2 + e^{3t}$

- Or, what if $f(t)$ has a constant in the front?

Like: $4y'' + 2y' + 5y = 5 \sin t$

- Or, what if $f(t)$ has addition or subtraction and constants in the front?

Like: $4y'' + 2y' + 5y = 3e^t - 2 \sin t$

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 1)

The Superposition Principle

1) If y_1 is a solution to $ay'' + by' + cy = f_1(t)$
   and y_2 is a solution to $ay'' + by' + cy = f_2(t)$
   then $y_1 + y_2$ is a solution to $ay'' + by' + cy = f_1(t) + f_2(t)$

2) If y_1 is a solution to $ay'' + by' + cy = f_1(t)$
   and k is a constant
   then ky_1 is a solution to $ay'' + by' + cy = kf_1(t)$

3) If y_1 is a solution to $ay'' + by' + cy = f_1(t)$
   and y_2 is a solution to $ay'' + by' + cy = f_2(t)$
   and k_1 and k_2 are constants
   then $k_1y_1 + k_2y_2$ is a solution to $ay'' + by' + cy = k_1f_1(t) + k_2f_2(t)$

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The Superposition Principle

Superposition Principle

Theorem 3. If y_1 is a solution to the differential equation

$$ay'' + by' + cy = f_1(t) ,$$

and y_2 is a solution to

$$ay'' + by' + cy = f_2(t) ,$$

then for any constants k_1 and k_2, the function $k_1y_1 + k_2y_2$ is a solution to the differential equation

$$ay'' + by' + cy = k_1f_1(t) + k_2f_2(t) .$$

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 1)

Example 1 Find a particular solution to

(1) $y'' + 3y' + 2y = 3t + 10e^{3t}$ and (2) $y'' + 3y' + 2y = -9t + 20e^{3t}$.

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Example 2 Given that $y_p(t) = t^2$ is a particular solution to $y'' - y = 2 - t^2$, find a general solution and a solution satisfying $y(0) = 1, y'(0) = 0$.

Section 4.5: The Superposition Principle and The Method of Undetermined Coefficients Revisited (Part 1)

Example 4 Find a particular solution to $y'' - y = 8te^t + 2e^t$.